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THE USE OF ONE- AND TWO-DIMENSIONAL  
HYDRODYNAMIC MACHINE CALCULATIONS  
IN HIGH EXPLOSIVE RESEARCH

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UCRL-14531  
Mathematics and  
Computers, UC-32  
TID-4500 (45th Ed.)

UNIVERSITY OF CALIFORNIA  
Lawrence Radiation Laboratory  
Livermore, California

AEC Contract No. W-7405-eng-48

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Mark L. Wilkins

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ABSTRACT

Hydrodynamic calculations in conjunction with experiments provide an effective method for obtaining otherwise inaccessible information on high explosives. The credibility of the calculational program (code) is established by comparison with the steady state solutions for certain detonation problems obtained by the method of characteristics.

The machine calculation is then applied to specific problems in one and two dimensions to explain some of the difficulties an experimenter would encounter in attempting to measure the pressure at a detonation front.

INTRODUCTION

The ability to obtain a complete hydrodynamic solution to a given problem allows the experimenter to investigate areas of high explosive research that would otherwise be inaccessible; for example, the low pressure equation of state work using exploding cylinders that is reported in (1). In addition, with a complete solution to a given problem available, the experimenter is better able to pose and to interpret simple experiments. Disagreement between the calculations and the experiment is an indication that the physical model is not adequate.

Finite difference equations using the Von Neumann Q method (2 and 3) afford the technique to solve the partial differential equations of hydrodynamics in one- and two-space dimensions and time.

CHECKS ON THE NUMERICAL TECHNIQUE

In order to use a hydrodynamic code in a quantitative manner it is first necessary to establish the confidence level of the numerical technique. For geometries in one-space variable and time, the method of characteristics provides a closed form solution for a Chapman-Jouguet detonation. It is then very readily shown that the numerical technique can reproduce the results to any desired accuracy (4). For two-space variables and time a closed form solution for a detonation is not mathematically possible; however, the steady state solution for a detonation in two-space variables can be obtained by the method of characteristics and a numerical integration scheme.

A fairly critical test of a code to calculate effects due to edge rarefactions is to compare a time-dependent code calculation with the solution by the method of characteristics.

A calculation was made of a detonation in plane geometry with a two-dimensional time-dependent hydrodynamic code, HEMP, (3). A steady state condition was noted by examining the parameters at a fixed distance behind the detonation front and observing that they did not change with time. In a two-dimensional steady state detonation, the rarefactions from the lateral edges extend into the fluid up to the detonation front itself. The time-dependent calculation showed that the steady state condition was approached from the outside to the inside of the fluid. Figure 1 shows a steady state detonation, as calculated by the method of characteristics (5), compared to a time-dependent calculation where the detonation has proceeded a distance equal to four times the original high explosive thickness. At a fixed distance behind the front, parameters were compared after a length of burn of three and four thickness units. They were also compared to parameters calculated by the method of characteristics. The agreement was to the fourth decimal place. These comparisons show that not only does the time-dependent code reach a steady state condition but it reaches the correct steady state condition. It can be seen from the figure that the time-dependent calculation has not yet reached a steady state for positions close to the front.

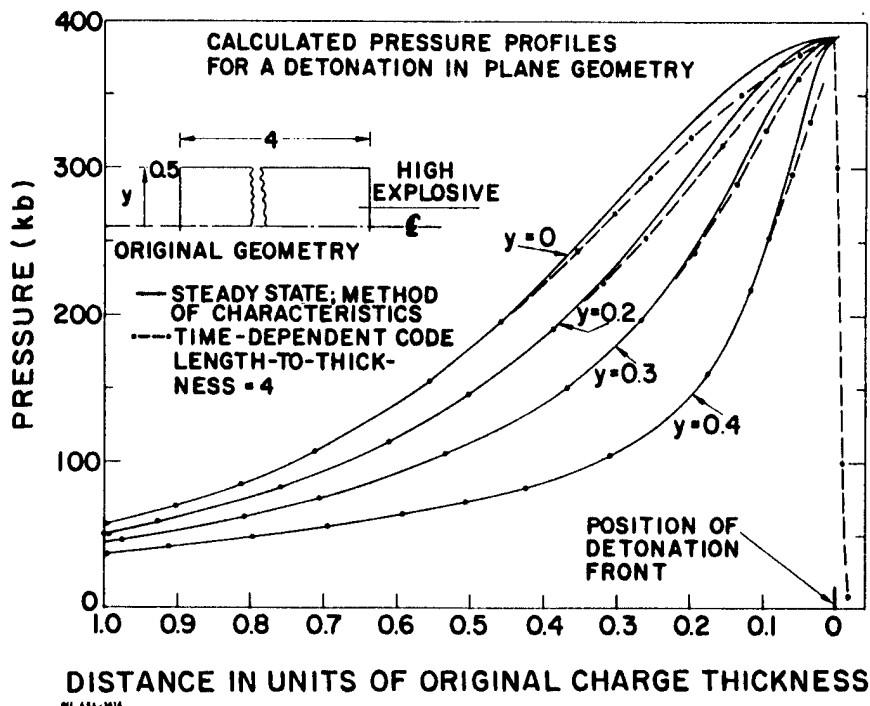


Fig. 1. Approach to steady state conditions for a time-dependent calculation. The outside profiles reach a steady state before the profile along the axis.

The calculations used Chapman-Jouguet (CJ), theory and a gamma law equation of state for the detonation product gases ( $\gamma = 2.6536$ ,  $P_{CJ} = 0.39$  mb, detonation velocity =  $0.88$  cm/ $\mu$ sec,  $\rho^0 = 1.84$  g/cc). The numerical results for the profiles shown in Fig. 1 by the method of characteristics are given in (5).

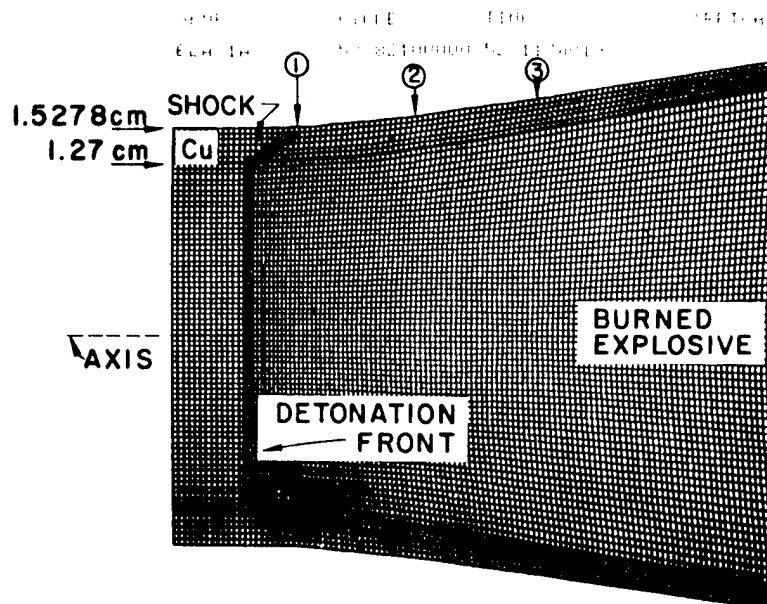
Figure 2a shows a code calculation of an exploding copper cylinder as is used in conjunction with experiment to determine the equation of state at low pressures for the detonation product gases of an explosive (1). The explosive used in this calculation was LX-04-1; the equation of state is given in (6). The calculation was allowed to proceed in time until a steady state condition was reached as noted by comparing parameters at a fixed distance behind the front for different lengths of run and noting that they did not change. This calculation was compared with the same problem done by Dr. N. E. Hoskin, AWRE, Aldermaston, England, who used the method of characteristics and included the shocks and rarefactions in the copper cylinder. The two calculations agreed, on the average, to better than  $10^{-2}$   $\mu$ sec for comparisons of the arrival times of the outside of the copper cylinder at a given radius. The numerical results are given in Table I.

Figure 2b shows the radius vs time history at a fixed position on the axis of the outside surface of the copper cylinder as would be seen by a slit camera (1).

Figure 2c is a schematic view showing the origin of the breaks in the cylinder, called out by 1, 2, and 3. A time-dependent calculation was also made with one-fourth the number of zones used for the calculation shown here, and the agreement was again to  $10^{-2}$   $\mu$ sec. These checks serve to establish that the numerical techniques used to solve the partial differential equations of hydrodynamics are sufficiently accurate such that the hydrodynamic code can be used with confidence for the analysis of hydrodynamic experiments.

#### APPLICATION TO DETONATION PRESSURE EXPERIMENTS

One of the main reasons for developing hydrodynamic codes is to be able to calculate the flow due to rarefactions. Calculations involving shocks and shock interactions can be made in a rather straightforward manner; it is the effect of rarefactions that cannot be readily calculated. It is the nature of hydrodynamics that rarefactions always overtake shocks. This effect can make experiments with high explosives very difficult to interpret. For example, consider the experimental problem of measuring a pressure at the detonation front after the detonation has reached a steady state. A convenient geometry would be to place a thin plate of metal at the end of an explosive charge that had a length-to-diameter ratio of about four and then to measure the initial front surface velocity for two thicknesses of the plate. In principle, the particle velocity at the high explosive metal interface can be determined by extrapolation from front surface velocity measurements for two thicknesses of metal; it is assumed that the relation between the front surface velocity and the particle velocity is known. With this information and the acoustic approximation (7), the pressure at the detonation front can be determined.



(a)

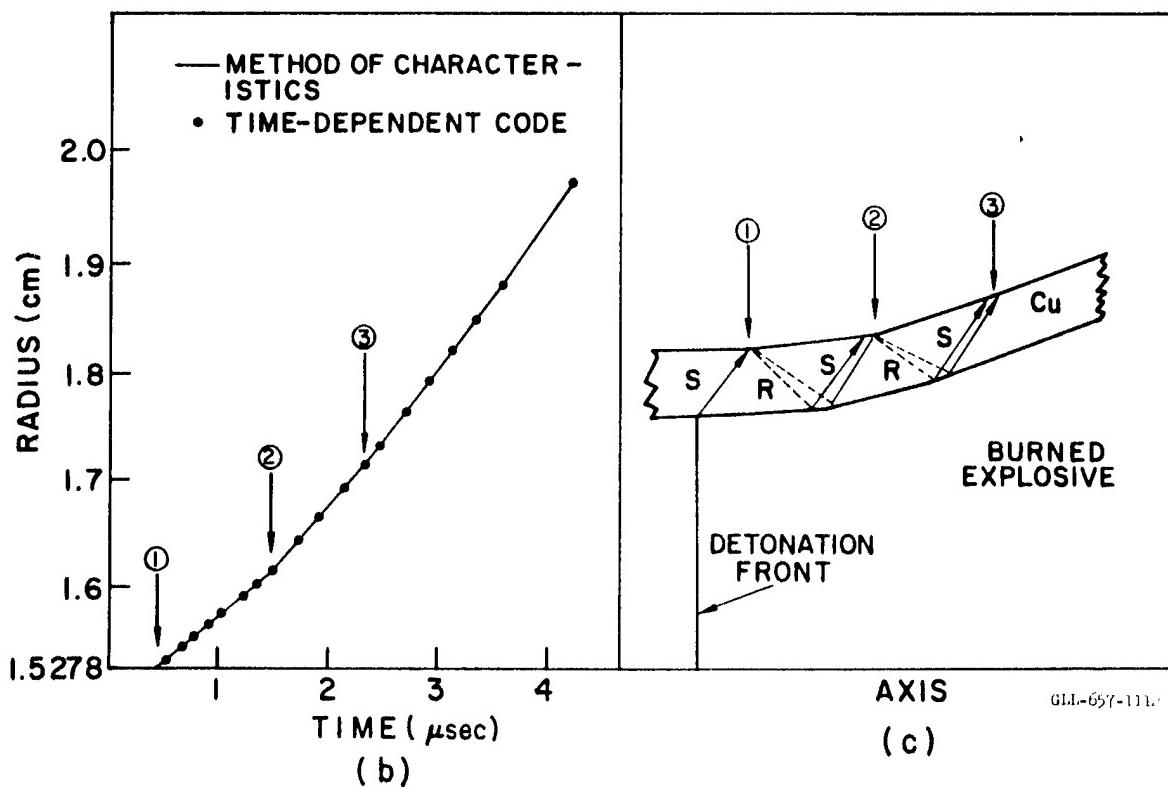


Fig. 2. (a). HEMP code calculation of an explosive charge detonated inside a copper cylinder.  
(b). Profile of the outside surface at a fixed position on the axis.  
(c). Schematic view showing the shocks, S, and the rarefactions, R.

Table I. Numerical values for the graph shown in Figure 2(b).

Radius $R^O = 15.278$ (mm)	Time by method of characteristics ( $\mu$ sec)	Time by HEMP code ( $\mu$ sec)
15.283	0.455	0.352
15.342	0.525	0.527
15.460	0.667	0.662
15.561	0.793	0.772
15.655	0.913	0.897
15.765	1.054	1.042
15.914	1.254	1.227
16.008	1.380	1.347
16.151	1.502	1.503
16.256	1.589	1.602
16.416	1.723	1.737
16.527	1.818	1.825
16.631	1.910	1.909
16.812	2.075	2.068
16.925	2.182	2.171
17.045	2.295	2.279
17.154	2.388	2.376
17.286	2.482	2.488
17.436	2.590	2.610
17.591	2.705	2.729
17.742	2.821	2.836
17.888	2.936	2.940
18.033	3.054	3.049
18.171	3.169	3.156
18.293	3.268	3.252
18.428	3.361	3.360
18.569	3.455	3.471
18.773	3.598	3.620
18.926	3.707	3.728
19.695	4.269	4.271

Some of the difficulties associated with this technique are discussed in the following paragraphs.

Figure 3 shows a calculation of a shock induced in an aluminum disc by a charge of explosive that had an original length-to-diameter ratio of four. For the cylindrical geometry used in this calculation a steady state condition was reached a little after three diameters of explosive burn. (For the plane geometry discussed earlier a steady state for positions just behind the front had not yet been attained after four lengths of explosive burn.) The pressure profiles in the explosive before striking the disc are shown in Fig. 4a.

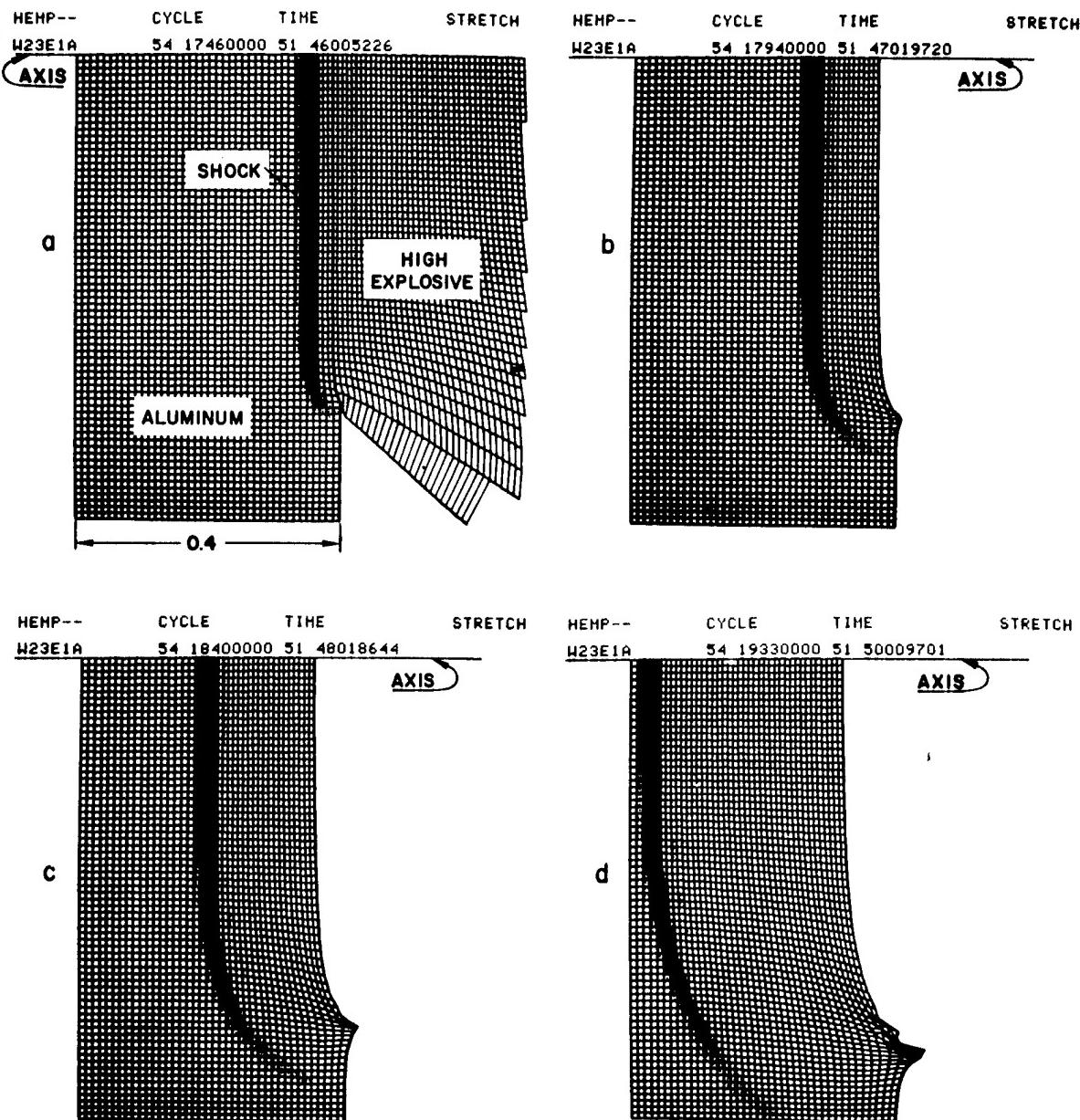


Fig. 3. HEMP code calculation of a cylindrical explosive charge (length to diameter = 4) in contact with an aluminum disc. For clarity the explosive is shown only for the first frame.

Figures 4b and 4c show the pressure profiles in the aluminum disc at three different times. It is noted that the peak pressure attenuates rapidly and that the pressure profiles steepen as the shock progresses into the disc. In one-space dimension the profiles would tend to become flatter rather than steeper as the shock wave progressed. The significance of these calculations is that the pressure wave attenuates very rapidly as it traverses the aluminum, due to the two-dimensional rarefactions present. The rapid drop in pressure will cause a tension wave to develop when the pressure wave reaches the free surface. The tension wave will greatly complicate the measurement

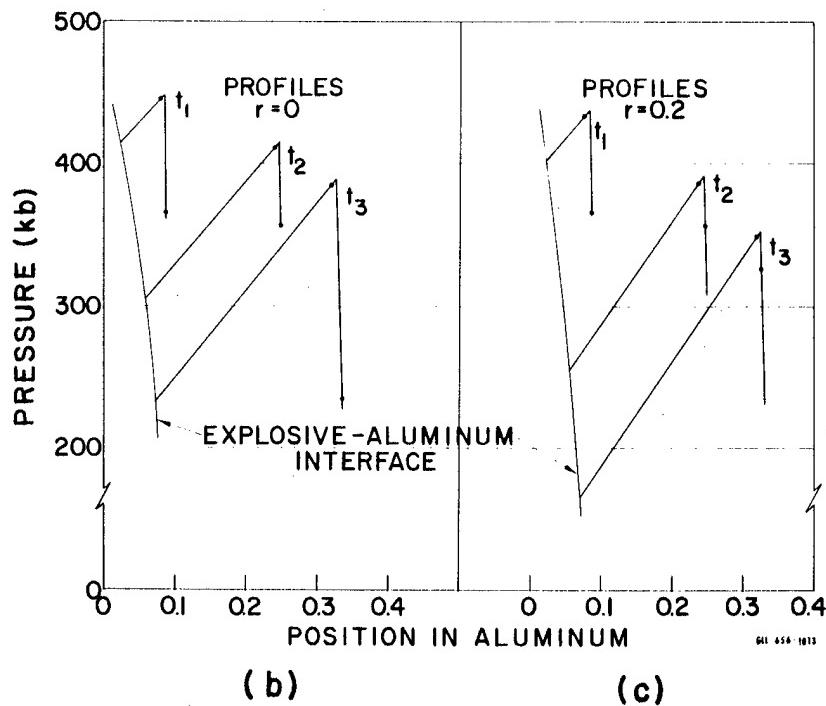
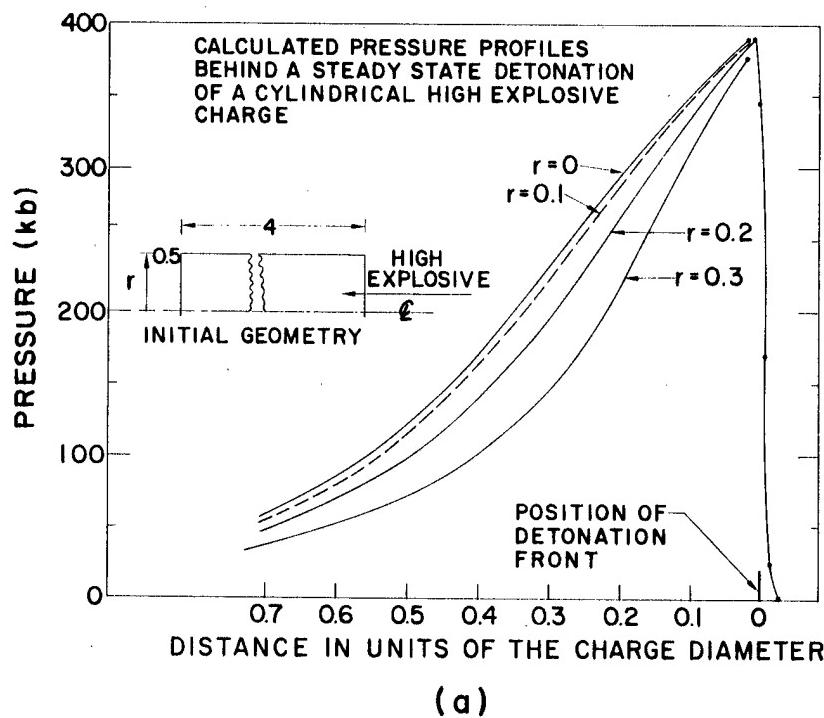


Fig. 4. (a). Steady state detonation calculated with a time-dependent hydrodynamic code. (b) and (c). Pressure profiles along the axis,  $r = 0$ , and off the axis,  $r = 0.2$ , in an aluminum disc placed on the end of the charge above (see Fig. 3).

of a front surface velocity by a distance time technique, since the material will decelerate during the measurement and, no doubt, will spall. Erroneous detonation pressures can be inferred from measurements if the experimenter is not fully aware of the severe effects of rarefactions.

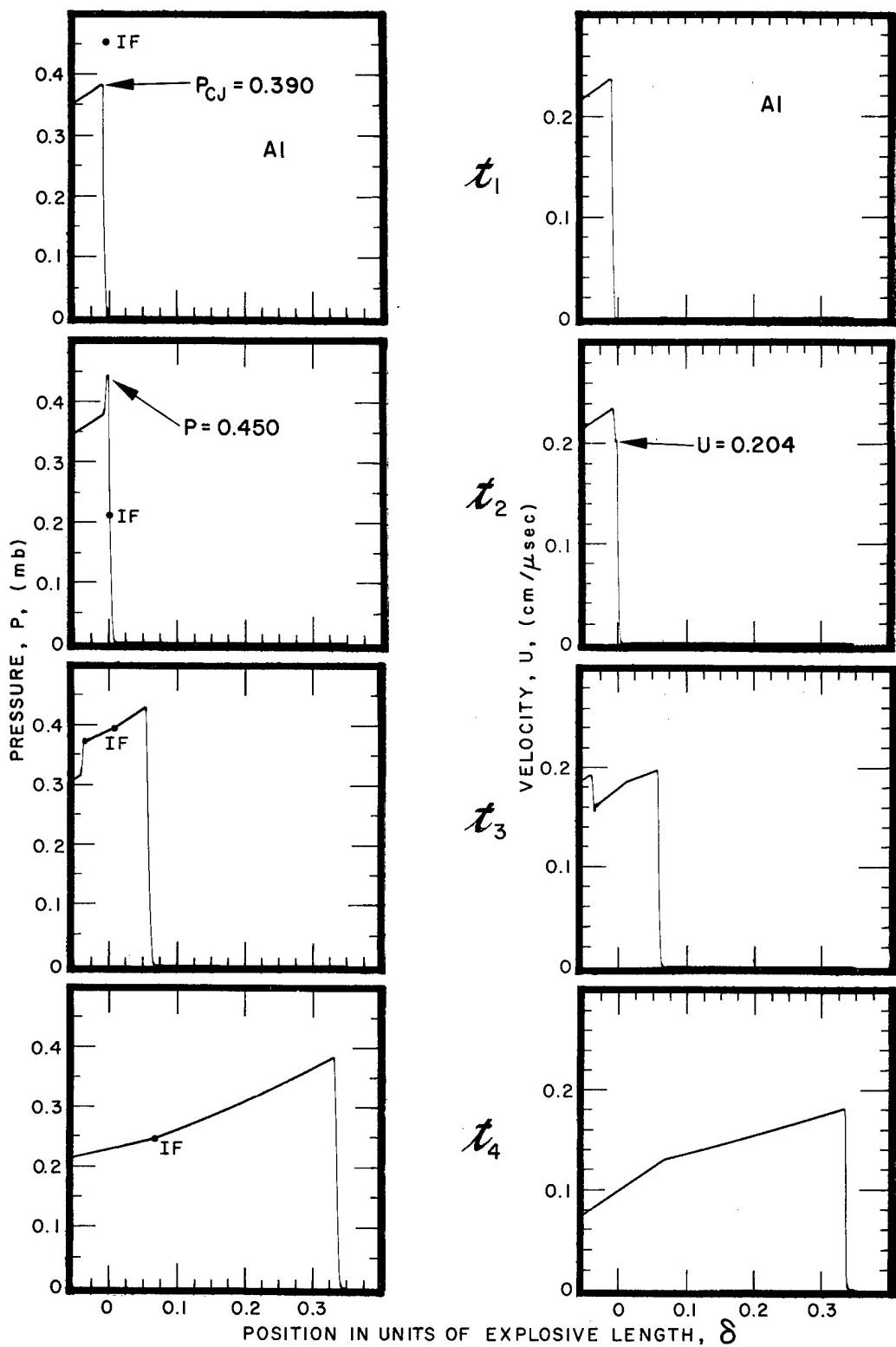
In Fig. 4 it is seen that distance scales with respect to the charge diameter. The pressure profile  $t_3$  in Fig. 4 will become  $t_1$  across the same physical plate thickness if the charge diameter is made larger. There will be two effects on a front surface velocity measurement: (1) the velocity corresponding to profile  $t_1$  will be greater than that due to profile  $t_3$  because the pressure attenuation is less; and (2) there is a much smaller pressure gradient across the fixed dimension plate for the profile  $t_1$ . Therefore, for the same base time measurement the effect of tensions will be less for profile  $t_1$  than for profile  $t_3$ .

If the detonation pressure is to be measured in this particular geometry, all the dimensions should be scaled by a factor that will allow sufficient time for the front surface velocity measurement to be made for a metal witness plate whose thickness is a small fraction of the high explosive diameter. However, the plate thickness must not be chosen so small that the effects of the reaction zone are still present.

Code calculations allow the experimenter to obtain "behind the scene" information and to better choose the parameters used for a specific experiment. A more suitable geometry for making detonation pressure measurements is to design a one-dimensional experiment, but even here the effects of rarefactions are very pronounced.

Figure 5 shows the results of a plane one-dimensional calculation for a geometry that could be used to determine a detonation pressure experimentally. An aluminum witness plate has been placed on one end of a unit length of high explosive. A gamma law equation of state was used for the detonation product gases ( $\gamma = 2.6536$ ,  $P_{CJ} = 0.39$  mb). The figures show the progress of the pressure wave and the material velocity as the shock traverses the aluminum. It can be seen that the peak transmitted pressure in the aluminum decays rapidly. This decay is due solely to the one-dimensional rarefaction behind the original detonation front. The physics model used here does not include a reaction zone in the explosive. The pressure wave in the aluminum would have additional structure if the reaction zone were included.

In Fig. 6 the pressure at the shock front is plotted as a function of the shock distance in the aluminum plate. A similar plot of the particle velocities would show the same shape. It is the particle velocity at essentially zero thickness that is required to determine a detonation pressure by the acoustic approximation (7). This velocity can be determined experimentally by measuring the front surface velocity for two thicknesses of metal and then extrapolating to find the material velocity at the high explosive metal interface. The usual assumption is that the particle velocity is one-half the free surface velocity where the free surface velocity is obtained by a distance-time experiment.



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Fig. 5. Explosive in contact with an aluminum plate. IF is interface position, and  $t$  is defined as follows :

- $t_1$  P-U in explosive before striking Al
- $t_2$  P-U just after explosive strikes plate
- $t_3$  Shocks in explosive and plate
- $t_4$  P and U at a late time.

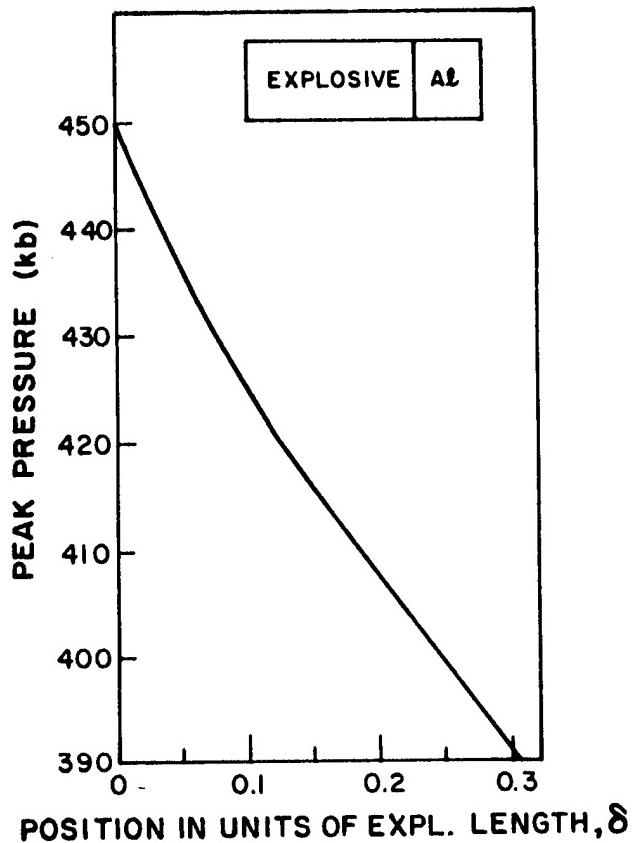


Fig. 6. Peak pressure,  $P$ , vs position,  $\delta$ , for the shock wave traversing the aluminum plate of Fig. 5.

With reference again to Fig. 6, it is seen that a linear extrapolation should not be attempted unless the experiment has been performed with the ratio of plate thickness to charge length less than about 0.1. If the two velocity measurements are made with ratios greater than this, the inferred detonation pressure will be too low since this curve is really not linear.

#### ACOUSTIC APPROXIMATION

As is well known, the acoustic approximation assumption is that the detonation products Hugoniot has a slope equal to  $-\rho^0 D$  at the metal Hugoniot point in the P-U plane. Chapman-Jouguet theory states that the slope of the Hugoniot is also equal to  $-\rho^0 D$  in the P-U plane at the detonation front. Therefore, for measurements made near the detonation pressure, where the effect of any curvature in the detonation products Hugoniot is small, the acoustic approximation is very nearly exact for a Chapman-Jouguet detonation.

At this point it is interesting to try out the acoustic approximation. For the problem shown in Fig. 5, the code calculation gives  $P = 0.450$  mb and  $U = 0.204$  cm/ $\mu$ sec for the values of the pressure and the particle velocity in the metal at the explosive interface. The detonation velocity and the initial density of the explosive are, respectively,  $D = 0.88$  cm/ $\mu$ sec and  $\rho^0 = 1.84$  g/cc.

Substitution into the acoustic approximation,  $P_d = 1/2 [P + \rho^0 D U]$ , will give the detonation pressure,  $P_d$ . The result of the substitution is  $P_d = 0.390$  mb, which is, in fact, the Chapman-Jouguet detonation pressure that was used in the calculation. This calculation provides a further check on the codes.

#### THE EFFECT OF TENSIONS

Usually, the particle velocity in the metal witness plate is obtained by a measurement of the time required for the metal front surface to traverse a given distance. The code calculations show that the front surface velocity is decelerated by a tension wave that originates when the shock wave reaches the front surface. A different average velocity will be recorded if the base distance of the measurement is changed (9). The hydrodynamic codes can be used to monitor the magnitude of the tension and the extent of its effect on the front surface velocity as a guide to the experimentalist.

When the calculation just described is repeated with magnesium as a witness plate instead of aluminum, the rate of decay for the pressure pulse is less. Consequently, the tension at the front surface is less for any given ratio of plate thickness to charge length. The use of magnesium as a witness plate allows a ratio of plate thickness to charge length a little higher than the 0.1 for aluminum, since the decay curve corresponding to Fig. 6 is not as rapid. Also, for measurements made in this ratio, there are no tension waves present for a reasonable time. The hydrodynamic reason for these effects is that the magnesium has a lower shock impedance than has this particular explosive, and a more flat-top wave is transmitted into the magnesium than is transmitted into the higher-impedance material, aluminum.

#### APPLICATION TO REACTION ZONE THICKNESS MEASUREMENTS

The reaction zone pressure and velocity distributions span a fixed distance in space. Therefore, scaling the explosive dimensions will have no effect on the reaction zone pressure profile, but will, of course, change the pressure profiles over fixed distances for the flow behind the detonation pressure. The very difficult problem of obtaining the correct reaction zone thickness by experimental front surface measurements on plates of varying thicknesses without the aid of a hydrodynamic code is discussed in the following paragraphs.

Figure 7 shows the calculated attenuation of the pressure in an aluminum plate in contact with a plausible reaction zone pressure profile. The other hydrodynamic parameters of the profile were chosen consistent with the Von Neumann detonation model. It is seen that the peak pressure decays at a decreasing rate as the pulse traverses the plate. Front surface velocity measurements for plates a few reaction zones in thickness will show a sharp drop in velocity due to the early rapid pressure decay. For measurements made on plates of increasing thicknesses the velocities will fall off more slowly because the rate of pressure decay is less. It is apparent from Fig. 7 that measurements made on a discrete number of plate thicknesses would show a break in a velocity-vs-thickness curve. This break in the curve might erroneously be interpreted as the end of the reaction zone. There will be another break in the velocity curve that corresponds to the Chapman-Jouguet state. However, the slow fade-out of the pulse will make this state difficult to determine and could very well lead to an assumed Chapman-Jouguet pressure that is too high.

The experimental problem is further complicated by tensions and possible spall. Thin plates have sharp pressure profiles that will cause large tensions. For thicker plates the profiles flatten out and the induced tension when they arrive at a front surface will be less.

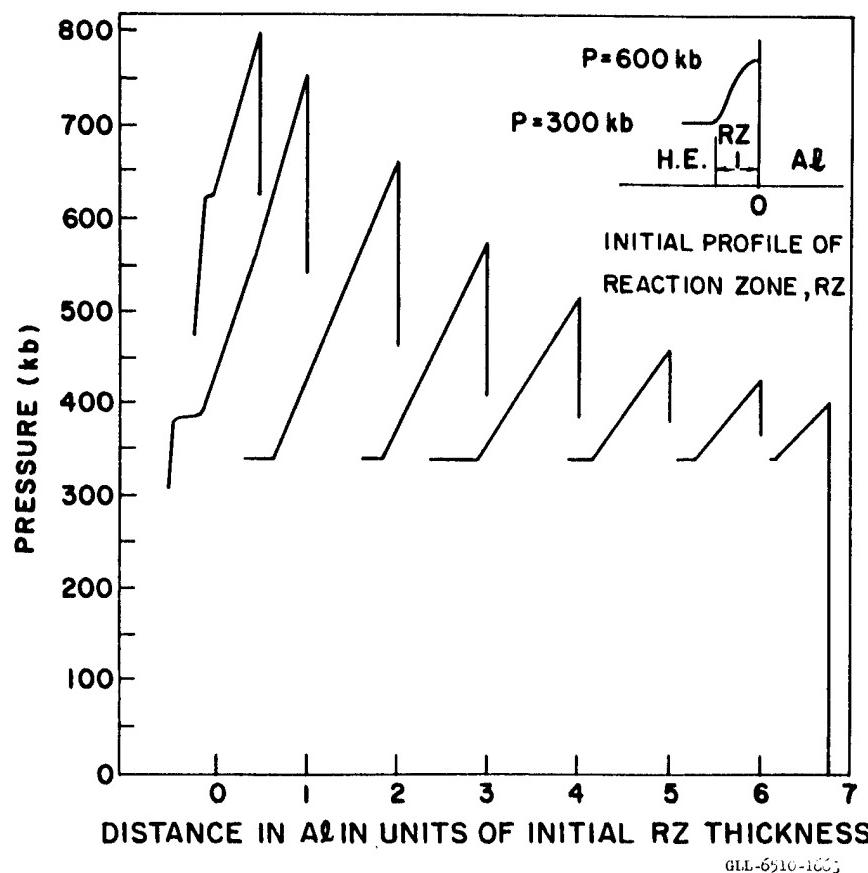


Fig. 7. Calculated pressure profiles in an aluminum plate induced by the reaction zone of an explosive. (The peak pressure decays rapidly at first and then at a slower rate as the profiles tend to flatten.)

A recent paper by A. N. Dremin (8) states for the same reasons as above that the properties of witness plates used in front surface velocity measurements result in a reaction zone that is too thin and in apparent breaks in the explosive pressure profile.

Interference of the reaction zone with a Chapman-Jouguet pressure determination can be avoided by choosing a plate thickness that is thought to be too large to show the effect of the pressure spike. A hydrodynamic code calculation can then be made where the detonation pressure is adjusted until the calculation matches the experimental measurement on the witness plate (6).

#### CONCLUSION

It has been shown that hydrodynamic codes are capable of accurately solving the equations of hydrodynamics. They, therefore, offer the means to perform calculations in conjunction with experiments and to use an iterative technique to obtain otherwise inaccessible data (1). In addition, they serve an important role in setting up and interpreting experiments in high explosive research.

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